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A new interpretation of Legendre's transformation and consequences

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Abstract

The Legendre transformation has found widespread application in thermodynamics, Hamilton-Lagrange-mechanics and optics. It attributes the values of the coordinates $(x, y(x))$ representing the points of a monotonic piecewise smooth functional curve $y(x)$ the slopes $m_x(x) = \frac{dy(x)}{dx}$ and the intercepts $\bar{y}(m_x)$ of their tangents on the $y-$axis. Thus, the initial curve $y(x)$ is represented by the ordered set of all slopes $m_x(x) = \frac{dy(x)}{dx}$ of its tangents together with their intercepts $\bar{y}(m_x)$ on the $y-$axis. It is shown that the transformed or conjugated function must basically be supplemented by a homogeneous linear function of the relevant variable. This is usually neglected in the literature. In addition, a new interpretation of the Legendre transformation is presented and discussed: For this purpose the derivative $m_x(x)$ is considered as the proper initial function and integrated between x_0 and x. This integral is complemented by the integral of $x(m_x)$ ((the inverse function of $m_x(x)$) over m_x between $m_{x0} = m(x_0)$ and $m_x = m_x(x)$, if $m_x(x)$ and $x(m_x)$ are strictly monotonic. The sum of both integrals yields the "area" $(xm_x - x_0 m_{x0})$. Legendre's transformation is obtained by reordering the respective terms. The procedure of transformation corresponds to integration by parts.

Some examples and consequences of the properties considered are demonstrated and discussed using the simple model of two-state systems. The general results of the present work remove possible internal inconsistencies in thermodynamics.